Lumpy Investment and Corporate Tax Policy

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Abstract

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1 Introduction

Corporate tax policy is an important instrument to influence firms' capital investment decisions and hence stimulate the economy. Its transmission channel is through either the user cost of capital according to the neoclassical theory of investment or Tobin's marginal Q according to the q-theory of investment.[1] The neoclassical theory assumes that firms do not face any investment frictions, while the q-theory takes into account of convex capital adjustment costs.

Recent empirical evidence documents that investment at the plant level is lumpy and infrequent, indicating the existence of nonconvex capital adjustment costs. Motived by this evidence, we address two central questions: (i) How does corporate tax policy affect investment at both the macro- and micro-levels in the presence of nonconvex capital adjustment costs? (ii) Is lumpy investment important quantitatively in determining the impact of tax policy on the economy in the short- and long-runs?

To answer these two questions, one has to overcome two major difficulties. First, in the presence of nonconvex adjustment costs, the standard q-theory widely used in tax policy may fail in the sense that investment may not be monotonically related to marginal Q (Caballero and Leahy (1996)). Even though a modified q-theory may work, the relationship between investment and marginal Q is nonlinear (Abel and Eberly (1994)). Second, investment at the micro-level is nonlinear, making aggregation difficult. This problem is even more severe in a dynamic general equilibrium context, because one has to deal with the cross-sectional distribution of firms when solving equilibrium prices.

Our solution to these two difficulties is based on the generalized (S,s) approach proposed by Caballero and Engle (1999). The key idea of this approach is to introduce stochastic fixed adjustment costs, which makes aggregation tractable. We combine this approach with the Abel and Eberly (1994) approach by assuming firms face both flow convex and nonconvex adjustment costs. As a result, at the micro-level, a modified q-theory applies in that investment is a nondecreasing function of marginal Q. In particular, there is a region for the stochastic fixed adjustment costs in which investment is zero, independent of marginal Q.

To resolve the curse of dimensionality issue encountered when numerically solving a general equilibrium model[2], Khan and Thomas (2003, 2008) and Bachmann et al (2008) use the Krusell and Smith (1998) algorithm, which approximates the distribution by a finite set of moments. Typically, the first moment is sufficient. We deal with the dimensionality problem by making

[1]See Tobin, Jorgenson and Hall and Jorgenson and Hayashi and Abel (1990).
[2]The cross sectional distribution of firms is a state variable.

two assumptions: (i) firms' production technology has constant returns to scale, (ii) fixed capital adjustment costs are proportional to the capital stock.[3] We show that under these two assumptions, the aggregate distribution matters only to the extent of its mean. We can then characterize equilibrium by a system of nonlinear difference equations, which can be tractably solved numerically.

Our model is based on Miao and Wang (2009). We extend Miao and Wang (2009) by incorporating tax policy. We study the short- and long-run effects of temporary and permanent changes in the corporate tax rate and the investment tax credit (ITC). We also analyze the impact of anticipation of tax changes. We find that in the presence of fixed adjustment costs, corporate tax policy affects a firm's decision on the timing and size of investment at the micro-level. At the macro-level, corporate tax policy affects the size of investment for each firm and the number of firms that make investment. Thus, corporate tax policy has both intensive and extensive margin effects. We numerically show that the extensive margin effect accounts for most of the impact of tax policy. The strength of this effect depends crucially on the cross-sectional distribution of firms. First, as Miao and Wang (2009) point out, the larger the steady-state elasticity of the adjustment rate with respect to the investment trigger, the larger is the extensive margin effect. Second, if the distribution is such that most firms have made capital adjustments prior to tax changes, then expansionary tax policy is less effective in stimulating the economy because the additional number of firms that decide to investment is constrained, while contractionary tax policy is more effective. The opposite is true if the distribution is such that most firms have not made capital adjustments prior to tax changes.

Our paper is related to a large literature on the impact of tax policy on investment beginning from Jorgenson (1967) and Hall and Jorgenson (1977) and Hall (1971). Most studies use a partial equilibrium framework (e.g., Auerbach (1989), Summers (1981), Abel (1982), and Hayashi (1980)). Instead, we use a general equlibrium nonstochastic growth model framework as in Chamely (1981) and Judd (1985, 1987).[4] Unlike these papers, we incorporate both convex and nonconvex capital adjustment costs as well as firm heterogeneity.

[3]Both assumptions seem innocuous. The first assumption is often used in the workhorse growth model. The second assumption has empirical support by the estimate in Cooper and Haltiwanger (2006).

[4]See Lujnqvist and Sargent (2008, Chapter 11) for a textbook treatment. See Auerbach and Kotlikoff (1987) for the study tax policy in the general equilibrium overlapping-generation framework.

2 The Model

We consider an infinite-horizon economy that consists of a representative household, a continuum of production units with a unit mass, and a government. Time is discrete and indexed by $t = 0, 1, 2, ...$ We identify a production unit with a firm or a plant. Firms are subject to idiosyncratic shocks to fixed capital adjustment costs. To focus on the dynamic effects of permanent and tempoarary tax changes, we abstract away from aggregate uncertainty and long-run economic growth. By a law of large numbers, all aggregate quantities and prices are deterministic over time.

2.1 Firms

All firms have identical production technology that combines labor and capital to produce output. Specifically, if firm j owns capital K_t^j and hires labor N_t^j, it produces output Y_t^j according to the production function:

$$Y_t^j = F\left(K_t^j, N_t^j\right), \tag{1}$$

Assume that F is strictly increasing, strictly concave, continuously differentiable, and satisfies the usual Inada conditions. In addition, it has constant returns to scale.

Each firm j may make investments I_t^j to increase its existing capital stock K_t^j. Investment incurs both nonconvex and convex adjustment costs. We follow Uzawa (1969) and Hayashi (1982) and introduce convex adjustment costs into the capital accumulation equation:

$$K_{t+1}^j = (1-\delta)K_t^j + K_t^j \Phi\left(\frac{I_t^j}{K_t^j}\right), \; K_0^j \text{ given}, \tag{2}$$

where δ is the depreciation rate and $\Phi(\cdot)$ is a strictly increasing, strictly concave and continuously differentiable function.[5] To facilitate analytical solutions, we follow Jermann (1998) and specify the convex adjustment cost function as:

$$\Phi(x) = \frac{\psi}{1-\theta} x^{1-\theta} + \varsigma, \tag{3}$$

where $\psi > 0$ and $\theta \in (0, 1)$. Nonconvex adjustment costs are fixed costs that must be paid if and only if the firm chooses to invest. As in Cooper and Haltiwanger (2006), we measure these costs

[5] An alternative way of modeling convex adjustment costs introduced by Lucas (1967), Gould (1968), and Treadway (1969) is to assume that these costs lower profits directly. Adopting this approach does not change the insights of our analysis.

as a fraction of the firm's capital stock.[6] That is, if firm j makes new investment, then it pays fixed costs $\xi_t^j K_t^j$, which is independent of the amount of investment. As will be clear later, this modeling of fixed costs is important to ensure that firm value is linearly homogenous. Following Caballero and Engel (1999), we assmue that ξ_t^j is identically and independently drawn from a distribution with density ϕ over $[0, \xi_{\max}]$ across firms and across time.

Each firm j pays dividends to households who are shareholders of the firm. Dividends are given by:

$$D_t^j = \left(1-\tau_t^k\right)\left(F(K_t^j, N_t^j) - w_t N_t^j\right) + \tau_t^k \delta K_t^j - \left(1-\tau_t^i\right) I_t^j - \xi_t^j K_t^j \mathbf{1}_{I_t^j \neq 0} \tag{4}$$

where w_t is the wage rate, τ_t^k is the corporate income tax rate, and τ_t^i is the investment tax credit (ITC). Note that depreciation is tax deductable.

After observing its idiosyncratic shock ξ_t^j, firm j's objective is to maximize cum-dividends market value of equity P_t^j:

$$\max \ P_t^j \equiv E_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} D_{t+s}^j, \tag{5}$$

subject to (2) and (4). Here, the expectation is taken with respect to the idiosyncractic shock distribution and $\beta^s \Lambda_{t+s}/\Lambda_t$ is the stochastic discount factor between period t and $t+s$. We will show later that Λ_{t+s} is a household's marginal utility in period $t+s$.

2.2 Households

All households are identical and derive utility from the consumption and labor sequences $\{C_t, N_t\}$ according to the time-additive utility function:

$$E\left[\sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right)\right], \tag{6}$$

where $\beta \in (0,1)$ is the discount factor, and U satisfies the usual assumptions. Each household chooses consumption C_t, labor supply N_t, and shareholdings α_{t+1}^j to maximize utility (6) subject to the budget constraint:

$$C_t + \int \alpha_{t+1}^j \left(P_t^j - D_t^j\right) dj = \int \alpha_t^j P_t^j dj + \left(1-\tau_t^n\right) w_t N_t + T_t, \tag{7}$$

[6] There are several ways to model fixed adjustment costs in the literature. Fixed costs may be proportional to the demand shock (Abel and Eberly (1998)), profits (Caballero and Engel (1999) and Cooper and Haltiwanger (2006)), or labor costs (Thomas (2002) and Khan and Thomas (2003, 2008)).

where τ_t^n is the labor income tax rate and T_t denotes government transfers (lump-sum taxes) if $T_t > (<) 0$. The first-order conditions are given by:

$$\Lambda_t \left(P_t^j - D_t^j \right) = \beta E_t \Lambda_{t+1} P_{t+1}^j, \tag{8}$$

$$U_1 \left(C_t, N_t \right) = \Lambda_t, \tag{9}$$

$$-U_2 \left(C_t, N_t \right) = \Lambda_t \left(1 - \tau_t^n \right) w_t. \tag{10}$$

Equations (8)-(9) imply that the stock price P_t^j is given by the discounted present value of dividends as in equation (5). In addition, Λ_t is equal to the marginal utility of consumption.

2.3 Government

The government finances government spending G_t by corporate and personal taxes. We assume lump-sum taxes (or transfers) are available so that the government budget is balanced. The government budget is given by:

$$G_t + T_t = \tau_t^k \int \left(F(K_t^j, N_t^j) - w_t N_t^j - \delta K_t^j \right) dj + \tau_t^n w_t N_t - \tau_t^i \int I_t^j dj, \tag{11}$$

where T_t represents lump-sum transfers (taxes) if $T_t > 0$ ($T_t < 0$).

2.4 Competitive Equilibrium

The sequences of quantities $\{I_t^j, N_t^j, K_t^j\}_{t \geq 0}$, $\{C_t, N_t\}_{t \geq 0}$, prices $\{w_t, P_t^j\}_{t \geq 0}$ for $j \in [0,1]$, and government policy $\left\{ \tau_t^k, \tau_t^n, \tau_t^i, T_t, G_t \right\}$ constitute a competitive equilibrium if the following conditions are satisfied:

(i) Given prices $\{w_t\}_{t \geq 0}$, $\{I_t^j, N_t^j\}_{t \geq 0}$ solves firm j's problem (5) subject to the law of motion (2).

(ii) Given prices $\left\{ w_t, P_t^j \right\}_{t \geq 0}$, $\{C_t, N_t, \alpha_{t+1}^j\}_{t \geq 0}$ maximizes utility in (6) subject to the budget constraint (7).

(iii) Markets clear in that:

$$\alpha_t^j = 1,$$

$$N_t = \int N_t^j dj,$$

$$C_t + \int I_t^j dj + \int \xi_t^j K_t^j \mathbf{1}_{I_t^j \neq 0} dj + G_t = \int F \left(K_t^j, N_t^j \right) dj. \tag{12}$$

(iv) The government budget constraint (11) is satisfied.

3 Equilibrium Properties

We start by analyzing a single firm's optimal investment policy, holding prices fixed. We then conduct aggregation and characterize equilibrium aggregate dynamics by a system of nonlinear difference equations.

3.1 Optimal Investment Policy

To simplify problem (5), we first solve a firm's static labor choice decision. Let $n_t^j = N_t^j / K_t^j$. The first-order condition with respect to labor yields:

$$f' \left(A_t n_t^j \right) A_t = w_t, \tag{13}$$

where we define $f(\cdot) = F(1, \cdot)$. This equation reveals that all firms choose the same labor-capital ratio in that $n_t^j = n_t = n(w_t, A_t)$ for all j. We can then derive firm j's operating profits:

$$\max_{N_t^j} F\left(K_t^j, A_t N_t^j\right) - w_t N_t^j = R_t K_t^j,$$

where $R_t = f(A_t n_t) - w_t n_t$ is independent of j. Note that R_t also represents the marginal product of capital because F has constant returns to scale. Let $i_t^j = I_t^j / K_t^j$ denote firm j's investment rate. We can then express dividends in (4) as:

$$D_t^j = \left[\left(1 - \tau_t^k\right) R_t + \tau_t^k \delta - \left(1 - \tau_t^i\right) i_t^j - \xi_t^j \mathbf{1}_{i_t^j \neq 0} \right] K_t^j,$$

and rewrite (2) as

$$K_{t+1}^j = \left[(1 - \delta) + \Phi(i_t^j) \right] K_t^j. \tag{14}$$

The above two equations imply that equity value or firm value are linear in capital K_t^j. We can then write firm value as $V_t^j K_t^j$ and rewrite problem (5) by dynamic programming:

$$V_t^j K_t^j = \max_{i_t^j} \left[\left(1 - \tau_t^k\right) R_t + \tau_t^k \delta - \left(1 - \tau_t^i\right) i_t^j - \xi_t^j \mathbf{1}_{i_t^j \neq 0} \right] K_t^j + E_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j K_{t+1}^j \right], \tag{15}$$

subject to (14). Substituting equation (14) into equation (15) yields:

$$V_t^j = \max_{i_t^j} \; \left(1 - \tau_t^k\right) R_t + \tau_t^k \delta - i_t^j - \xi_t^j \mathbf{1}_{i_t^j \neq 0} + g(i_t^j) E_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}^j \right], \tag{16}$$

where we define:

$$g(i_t^j) = 1 - \delta + \Phi(i_t^j). \tag{17}$$

Since V_t^j depends on ξ_t^j, we write it as $V_t^j = V_t\left(\xi_t^j\right)$ for some function V_t and suppress its dependence on other variables. We aggregate each firm's price of capital V_t^j and define the aggregate value of the firm per unit of capital as:

$$\bar{V}_t = \int_0^{\xi_{\max}} V_t\left(\xi\right)\phi(\xi)d\xi. \tag{18}$$

Because ξ_t^j is iid across both time and firms and there is no aggregate shock, we obtain:

$$E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}V_{t+1}^j\right] = \frac{\Lambda_{t+1}}{\Lambda_t}\int_0^{\xi_{\max}} V_t\left(\xi\right)\phi(\xi)d\xi = \frac{\Lambda_{t+1}}{\Lambda_t}\bar{V}_{t+1}, \tag{19}$$

After defining (aggregate) marginal Q as the discounted shadow value of a marginal unit of investment:

$$Q_t = \frac{\beta\Lambda_{t+1}}{\Lambda_t}\bar{V}_{t+1}. \tag{20}$$

we can rewrite problem (16) as:

$$V_t\left(\xi_t^j\right) = \max_{i_t^j}\ \left(1-\tau_t^k\right)R_t + \tau_t^k\delta - \left(1-\tau_t^i\right)i_t^j - \xi_t^j\mathbf{1}_{i_t^j\neq 0} + g(i_t^j)Q_t. \tag{21}$$

From this problem, we can characterize a firm's optimal investment policy by a generalized (S,s) rule (Caballero and Engel (1999)).

Proposition 1 *Firm j's optimal investment policy is characterized by the (S,s) policy in that there is a unique trigger value $\bar{\xi}_t > 0$ such that the firm invests if and only if $\xi_t^j \leq \bar{\xi}_t \equiv \min\{\xi_t^*, \xi_{\max}\}$, where the cutoff value ξ_t^* satisfies the equation:*

$$\frac{\theta}{1-\theta}\left(\psi Q_t\right)^{\frac{1}{\theta}}\left(1-\tau_t^i\right)^{\frac{\theta-1}{\theta}} = \xi_t^*. \tag{22}$$

The optimal target investment level is given by:

$$i_t^j = \left(\frac{\psi Q_t}{1-\tau_t^i}\right)^{\frac{1}{\theta}}. \tag{23}$$

Marginal Q satisfies:

$$Q_t = \frac{\beta\Lambda_{t+1}}{\Lambda_t}\left\{\ \left(1-\tau_{t+1}^k\right)R_{t+1} + \tau_{t+1}^k\delta + \left(1-\delta+\varsigma\right)Q_{t+1} + \int_0^{\bar{\xi}_t}\left[\xi_{t+1}^* - \xi\right]\phi(\xi)d\xi\right\}. \tag{24}$$

Equation (22) says that, at the value ξ_t^*, the benefit from investment is equal to the fixed cost of investment so that the firm is indifferent. It is possible that ξ_t^* exceeds the upper support of the fixed costs. In this case, we set the investment trigger $\bar{\xi}_t = \xi_{\max}$.

Equation (23) shows that the optimal investment level is independent of a firm's characteristics such as its capital or idiosyncratic shock. It is positively related to marginal Q if and only if the firm's idiosyncratic fixed cost shock ξ_t^j is lower than the trigger value $\bar{\xi}_t$, conditioned on the aggregate state and tax policy in the economy. When $\xi_t^j > \bar{\xi}_t$, firm j chooses not to invest. This zero investment is unrelated to marginal Q. As a result, optimal investment may not be related to marginal Q in the presence of fixed adjustment costs, a point made by Caballero and Leahy (1996). But it is still a nondecreasing function of marginal Q. Thus, a modified q-theory still works (Abel and Eberly (1994)).

Equation (24) is an asset-pricing equation which states that the aggregate marginal Q is equal to the present value of marginal product of capital, plus an option value of waiting because of the fixed adjustment costs. When the shock $\xi_t^j > \bar{\xi}_t$, it is not optimal to pay the fixed costs to make investment. Firms will wait to invest until $\xi_t^j \leq \bar{\xi}_t$ and there is an option value of waiting.

We should emphasize that the the investment trigger $\bar{\xi}_t$ depends on the aggregate capital stock and corporate tax policy. Thus, corporate tax policy affects the timing of investment at the micro-level in that it affects the *adjustment hazard*, $\int_0^{\bar{\xi}_t} \phi(\xi)\, d\xi$. Under the q-theory of investment, corporate tax policy has only an intensive margin effect on investment in the sense that it can affect how much a firm may invest, but cannot affect how how likely a firm may investment.

3.2 Aggregation and Equilibrium Characterization

Given the linear homogeneity feature of firm value, we can conduct aggregation tractably. We define aggregate capital $K_t = \int K_t^j dj$, aggregate labor demand $N_t = \int N_t^j dj$, aggregate output $Y_t = \int Y_t^j dj$, and aggregate investment expenditure in consumption units $I_t = \int I_t^j dj$.

Proposition 2 *The aggregate equilibrium sequences* $\{Y_t,\, N_t,\, C_t,\, I_t,\, K_{t+1}, Q_t, \bar{\xi}_t\}_{t\geq 0}$ *are characterized by the following system of difference equations:*

$$I_t/K_t = \left(\frac{\psi Q_t}{1-\tau_t^i}\right)^{\frac{1}{\theta}} \int_0^{\bar{\xi}_t} \phi(\xi) d\xi, \tag{25}$$

$$K_{t+1} = (1-\delta+\varsigma)K_t + \frac{\psi}{1-\theta} K_t \left(I_t/K_t\right)^{1-\theta} \left[\int_0^{\bar{\xi}_t} \phi(\xi) d\xi\right]^{\theta}, \tag{26}$$

$$Y_t = F\left(K_t, N_t\right) = G_t + I_t + C_t + K_t \int_0^{\bar{\xi}_t} \xi \phi(\xi) d\xi, \tag{27}$$

$$\frac{-U_2\left(C_t,N_t\right)}{U_1\left(C_t,N_t\right)}=\left(1-\tau_t^n\right)F_2\left(K_t,N_t\right), \tag{28}$$

$$\begin{aligned} Q_t &= \frac{\beta U_1\left(C_{t+1},N_{t+1}\right)}{U_1\left(C_t,N_t\right)}\left[\left(1-\tau_{t+1}^k\right)F_1\left(K_{t+1},N_{t+1}\right)+\tau_{t+1}^k\delta+\left(1-\delta+\varsigma\right)Q_{t+1}\right. \\ &\left.+\int_0^{\bar{\xi}_{t+1}}\left(\xi_{t+1}^*-\xi\right)\phi(\xi)d\xi\right]. \end{aligned} \tag{29}$$

where $\bar{\xi}_t$ *and* ξ_t^* *are given in Proposition 1.*

We derive equations (25) and (26) by aggregating equations (2) and (23). Equation (25) reveals the aggregate investment rate is equal to a firm's target investment rate multiplied by the fraction of adjusting firms (or the adjustment rate), $\int_0^{\bar{\xi}_t}\xi\phi(\xi)d\xi$. Thus, corporate tax policy has both intensive and extensive margin effects on investment in the presence of fixed adjustment costs. To see the magnitude of these effects, we log-linearize equation (25) to obtain:

$$\hat{I}_t-\hat{K}_t=\underbrace{\frac{1}{\theta}\left(\hat{Q}_t+\frac{\tau^i}{1-\tau^i}\hat{\tau}_t^i\right)}_{\text{intensive}}+\underbrace{\frac{\bar{\xi}\phi(\bar{\xi})}{\int_0^{\bar{\xi}}\phi(\xi)d\xi}\hat{\bar{\xi}}_t}_{\text{extensive}}, \tag{30}$$

The first term on the right side captures the usual intensive marginal effect in the q-theory of investment without nonconvex adjustment costs. A change in τ_t^k or τ_t^i affects the price of capital Q as revealed by asset-pricing equation (29), and thus induces firms to decide how much to investment. The second term on the right-side of equation (30) represents the extensive margin effect. A change in τ_t^k or τ_t^i and the resulting change in Q also affects a firm's decison on when to invest. Thus, it affects the adjustment trigger $\bar{\xi}_t$ and hence the number of firms to make investment. The extent of this extensive margin effect is measured by the coefficient of $\hat{\bar{\xi}}_t$, which is the steady-state elasticity of the adjustment rate with respect to the adjustment trigger $\bar{\xi}$. The larger the elasticity, the larger is the intensive margin effect.

3.3 Steady State

We consider a deterministic steady state in which goverment policy variables $\left(\tau_t^k,\tau_t^n,\tau_t^i,T_t,G_t\right)$ stay constant over time. In addition, all aggregate equilibrium quantities, prices and the investment trigger are constant over time, while at the firm-level firms still face idiosyncratic fixed costs shocks. The following proposition characterizes the steady-state aggregate variables $\left(Y,C,N,K,I,Q,\bar{\xi}\right)$, where $\bar{\xi}$ is an interior solution.

Proposition 3 *Suppose*

$$0 < \delta - \varsigma < \frac{\psi}{(1-\theta)^{\theta}\,\theta^{(1-\theta)}} \left(\frac{\xi_{\max}}{1-\tau^i}\right)^{1-\theta} \int_0^{\xi_{\max}} \phi(\xi) d\xi.$$

Then the steady-state investment trigger $\bar{\xi} \in (0, \xi_{\max})$ is the unique solution to the equation:

$$\delta - \varsigma = \frac{\psi}{(1-\theta)^{\theta}\,\theta^{(1-\theta)}} \left(\frac{\bar{\xi}}{1-\tau^i}\right)^{1-\theta} \int_0^{\bar{\xi}} \phi(\xi) d\xi. \tag{31}$$

Given this value $\bar{\xi}$, the steady-state value of Q is given by:

$$Q = \frac{1}{\psi} \left(\frac{\bar{\xi}\,(1-\theta)}{\theta}\right)^{\theta} \left(1-\tau^i\right)^{1-\theta}. \tag{32}$$

The other steady-state values (I, K, C, N) satisfy:

$$\frac{I}{K} = \frac{Q}{1-\tau^i} (\delta - \varsigma) (1-\theta), \tag{33}$$

$$F(K, N) = I + C + K \int_0^{\bar{\xi}} \xi \phi(\xi)\, d\xi + G, \tag{34}$$

$$\frac{-U_2(C, N)}{U_1(C, N)} = (1-\tau^n) F_2(K, N), \tag{35}$$

$$Q = \frac{\beta}{1-\beta(1-\delta+\varsigma)} \left\{ \left(1-\tau^k\right) F_1(K, N) + \tau^k \delta + \int_0^{\bar{\xi}} (\xi^* - \xi)\, \phi(\xi) d\xi \right\}. \tag{36}$$

This proposition shows that the steady state investment trigger is independent of the corporate tax rate τ^k, but it decreases with the ITC τ^i. In the steady state, changes in τ^k have an intensive margin effect by affecting Q and the target investment rate, but do not have an extensitive margin effect by affecting the adjustment rate. Changes in τ^i have both positive intensive and extensive margin effect. From Proposition 3, it is straightforward to prove that, in the steady state, the adjustment rate and marginal Q decreases with τ^i, while the investment rate increases with τ^i.

4 Numerical Results

We evaluate our lumpy investment model quantitatively and compare this model with two benchmark models. The first one is a *frictionless RBC model*, obtained by removing all adjustment costs in the model presented in Section 2. In particular, we set $\xi_t^j = \theta = \varsigma = 0$ and $\psi = 1$. The second one is obtained by removing fixed adjustment costs only ($\xi_t^j = 0$). We call this model *partial adjustment model*. In both benchmark models, all firms make identical decisions and thus they give the same aggregate equilibrium allocations as that in a standard representative-agent and a representative-firm RBC model. Because we can characterize the equilibria for all three models by systems of nonlinear difference equations, we can solve them numerically using standard methods (e.g., log-linear approximation method). To do so, we need first to calibrate the models.

4.1 Baseline Parametrization

For all model economies, we take the Cobb-Douglas production function, $F(K,N) = K^{\alpha}N^{1-\alpha}$, and the period utility function, $U(C,N) = \log(C) - \varphi N$, where $\varphi > 0$ is a parameter. We fix the length of period to correspond to one year, as in Thomas (2002), and Khan and Thomas (2003, 2008). Annual frequency allows us to use empirical evidence on establishment-level investment in selecting parameters for the fixed adjustment costs.

We first choose parameter values for preferences and technology to ensure that the steady-state of the frictionless RBC model is consistent with the long-run values of key postwar U.S. aggregates. Specifically, we set the subjective discount factor to $\beta = 0.96$, so that the implied real annual interest rate is 4% (Cooley and Prescott (1995)). We choose the value of φ in the utility function so that the steady-state hours are about 1/3 of available time spent in market work. We set the capital share $\alpha = 0.36$, implying a labor share of 0.64, which is close to the labor income share in the NIPA. We take the depreciate rate $\delta = 0.1$, as in the literature on business cycles (e.g., Prescott (1986)).

In a steady state, all tax rates are constant over time. By the estimates from McGrattan and Prescott (2005) and Prescott (2002), we set capital tax rate $\tau^k = 0.35$ and labor tax rate $\tau^n = 0.32$. Because the investment tax credit is typically used as a short-run stimulative policy, we set $\tau^i = 0$ in the steady state. We assume that the government spending G_t is constant over time and equal to 0.2 percent of the steady-state output in the frictionless RBC model. We fix this level of government spending for all experiments below.

It is often argued that convex adjustment costs are not observable directly and hence cannot be calibrated based on average data over the long run (e.g., Greenwood et al. (2000)). Thus, we impose the two restrictions, $\psi = \delta^{\theta}$ and $\varsigma = -\theta\delta/(1-\theta)$, so that the partial adjustment model and the frictionless RBC model give identical steady-state allocations.[7] Following Kiyotaki and West (1996), Thomas (2002), and Khan and Thomas (2003), we set $\theta = 1/5.98$, implying that

[7] Under the log-linear approximation method, only the curvature parameter θ in the convex adjustment cost function matters for the approximated equilibrium dynamics.

the Q-elasticity of the investment rate is 5.98.

We adopt the power function distribution for the idiosyncratic fixed cost shock with density $\phi(\xi) = \eta\xi^{\eta-1}/(\xi_{\max})^{\eta}$, $\eta > 0$. We need to calibrate two parameters $\xi_{\max}$ and η. We select values such that our model's steady-state cross-sectional statistics match micro-level evidence on the investment lumpiness reported by Cooper and Haltiwanger (2006). Cooper and Haltiwanger (2006) find that the inaction rate is 0.081 and the positive spike rate is about 0.186. A positive investment spike is defined as the investment rate exceeding 0.2. For the power function distribution, the steady state inaction rate are given by $1 - \left(\bar{\xi}/\xi_{\max}\right)^{\eta}$ and the steady-state investment rate is given by equation (33). Here $\bar{\xi}$ is the steady-state trigger value determined in equation (31). Because our model implies that the target investment rate is identical for all firms, our model cannot match the spike rate observed in the data. Therefore, there are many combinations of η and $\xi_{\max}$ to match the inaction rate of 0.081. As baseline values, we follow Khan and Thomas (2003, 2008) and take a uniform distribution ($\eta = 1$). This implies that $\xi_{\max} = 0.0242$. In this case, we can compute that in the steady state, total fixed adjustment costs account for 2 percent of output, 10 percent of total investment spending, and 1 percent of the aggregate capital stock, which are reasonable according to the estimation by Cooper and Haltiwanger (2006).

We summarize the baseline parameter values in Table 1.

Table 1. Baseline Parameter Vales

β	φ	α	δ	τ^k	τ^n	τ^i	G	θ	$\xi_{\max}$	η
0.9615	2.5843	0.36	0.1	0.35	0.32	0	0.105	1/5.98	0.0242	1

We suppose the economy in period 1 is in the initial steady state with parameter values given in Table 1. We then consider the economy's responses to changes in corporate tax policy by changing sequences of tax rates $\left\{\tau_t^k\right\}$ and ITC $\left\{\tau_t^i\right\}$. We hold labor income tax rates τ^n constant at the value in Table 1 and allow lump-sum taxes to adjust to balance government budget. We consider four calibrated models denoted by RBC, PA, Lumpy1, and Lumpy 2. The first three models refer to the frictionless RBC, partial adjustment, and the lumpy investment models with the parameter values given in Table 1. "Lumpy2" refers to the lumpy investment model with parameter values given in Table 1 except that we set $\eta = 10$ and recalibrate $\xi_{\max}$ to match the inaction rate of 0.081.[8] We use the Lumpy2 model to illustrate the importance of the extensive margin effect. Because the steady-state elasticity of the adjustment rate with respect to the investment trigger is equal to η for the power function distribution, the Lumpy2

[8]It is equal to 0.0224.

model should deliver a larger extensive margin effect than the Lumpy1 model by our analysis in Section 3.2.

4.2 Temporary Changes in the Corporate Tax Rate

We start with the first policy experiment in which the corporate tax rate τ_t^k decreases by 10 percentage point temporarily and this decrease lasts for only 4 periods. After this decrease, τ_t^k reverts to its previous level. Suppose this tax policy is unexpected, but once it occurs in period 1, the public has perfect foresight about the future tax rates.

Figure 1 presents the transition dynamics for four models (RBC, PA, Lumpy1 and Lumpy2) following this tax policy. We find these four models display similar transition dynamics. Specifically, a decrease of τ_t^k raises the price (Q) of capital immediately, leading to a jump of investment in the initial period. The price of capital starts to decrease until period 4 and then gradually rises to its steady state value because τ_t^k rises to its original level permanently starting in period 5. Consequently, investment follows a similar path, but consumption follows an opposite pattern. Given our adopted utility function, the wage rate w_t satisfies $aC_t = (1 - \tau^n) w_t$. Thus, wages and consumption must follow identical dynamics, leading to the labor hours follow a pattern opposite consumption because marginal product labor equals the after tax wage. Output rises on impact because labor rises and capital is predetemined. After period 1, output gradually decreases to its steady state value.

We define the after-tax (gross) interest rate as

$$r_{t+1} = \frac{U_1(C_t, N_t)}{\beta U_1(C_{t+1}, N_{t+1})} = \frac{C_{t+1}}{\beta C_t}.$$

Because the after-tax interest rate is proportional to consumption growth, it rises on impact and then decreases until period 4. After period 4, it gradually rises to its steady state value.

Figure 1 reveals that the short-run effect on investment and output for the PA model is smaller than for the RBC model because of the convex capital adjustment costs. The presence of fixed capital adjustment costs in the lumpy investment model makes the short-run impact of tax changes larger. The reason is that tax policy has an additional extensive margin effect as discussed in Section 3.2. Figure 1 shows that the adjustment rate rises by about 4 percent in period 1 for the Lumpy1 model. The total impact increase in the investment rate for the Lumpy1 model is about 7 percent, implying that the intensive margin effect contributes to about 3 percent of the increase. As discussed in Section 3.2, the larger the steady-state elasticity of the adjustment rate with respect to the investment trigger, the larger is the extensive margin effect. The transition dynamics for the Lumpy2 model displayed in Figure 1 illustrate this point. The adjustment rate rises by about 8.4 percent on impact, which contributes to almost all of the increase (about 10 percent) in the investment rate.

Despite the large extensive margin effect, the dynamic responses of the tax change for the lumpy investment model are similar to those for the RBC model and the partial adjustment model. The reason is that the general equilibrium price movements smooth aggregate investment dynamics.[9] Figure 2 illustrates this point by presenting transition dynamics for the partial adjustment and lumpy investment models in partial equilibrium. In particular, we fix the interest rate and wage rate at the steady state values through the transition period. In response to the tax decrease, the price of capital Q rises by 1.7, 2.2, 2.5 percent for the PA, Lumpy1, and Lumpy2 models, respectively, much higher than the corresponding values, 0.74, 0.59, 0.16 in general equilibrium. As a result, the increase in the adjustment rate in partial equilibrium is much higher than that in general equilibrium. In particular, the increase in Q is so high that all firms decide to make capital adjustments in the first two periods for the Lumpy1 model and in the first three periods for the Lumpy2 model. This large extensive margin effect causes the aggregate investment rate rises by about 21 and 22 percent for the Lumpy1 and Lumpy2 models, respectively. This increase is much larger than the corresponding increase of 7.2 and 10 percent in general equilibrium.

We also emphasize that in a partial equilibrium model with competitive firms and constant returns to scale technology, Q can be determined independent of capital. This can be seen from equation (29), in which the marginal revenue product of capital R_t is constant when the wage rate is fixed. Thus, a temporary change in τ_t^k can have a permanent effect on the capital stock, as revealed in Figure 2 (See Abel (1982, ??)). This result is in sharp contrast to that in general equilibrium, suggesting that a partial equilibrium analysis of tax policy can be quite misleading.

We next turn to the case in which the temporary decrease in τ_t^k is anticipated initially to be effective in period 5 and lasts for 4 periods. Figure 3 presents the transition dynamics. Anticipating the tax decrease in the future, firms react by raising investment immediately. In addition, the adjustment rate also rises immediately. The investment rate and the adjustment rate continue to rise until period 4. The household reacts by reducing consumption and raising labor supply immediately. From period 1 to period 4, consumption and hours gradually rise. Output also rises from periods 1 to 4. Starting from period 5, the economy's response is similar to that in the case of unexpected tax change presented in Figure 1. Comparing Figure 3 with

[9] Thomas (2002) first makes this point. Using various different numerical solution methods, Khan and Thomas (2003, 2008) and Miao and Wang (2009) confirm her finding.

Figure 1, we also find that the impact effects when the tax decrease is expected are smaller than when it is unexpected.

4.3 Permanent Changes in the Corporate Tax Rate

Figure 4 presents transition dynamics for the case in which the decrease in the corporate tax rate is permanent but unexpected initially. The economy's immediate response in this case is qualitatively similar to that in the case of unexpected temporary corporate tax cut. However, the steady state after a permenant tax cut is different than that before the tax cut, while the steady state does not change after a temporary tax cut. In addition, the transition paths following an unexpected permanent tax cut are monotonic, rather than nonmonotonic.

After an unexpected permanent 10 percentage cut in τ_t^k, it takes about 40 periods for the economy to reach a new steady state. The steady-state capital stock increases by 8.9, 8.9, 10.5, and 11.4 percent for the RBC, PA, Lumpy1 and Lumpy2 models, respectively. Corresponding to these four models, the steady-state output increases by 3.7, 3.7, 4.3, and 4.7, respectively, and the steady-state consumption increases by 2.9, 2.9, 3.5, 3.8, respectively. Due to the extensive margin effect, the presence of fixed capital adjustment costs make the steady-state effect on the economy larger.

We emphasize that the impact of tax policy depends on the initial distribution of firms. In our baseline calibration, 91.9 percent of firms have made capital adjustment in every period. It leaves less room for more firms to make adjustments in response to a decrease in τ^k. Figure 4 reveals that the investment trigger hits the upper support of the fixed costs from periods 1-4 for the Lumpy2 model, implying that the extensive margin effect is constrained. If the initial adjustment rate is small, more firms will respond to a capital tax decrease, making the extensive margin effect large. To illustrate this point, we conduct an experiment with the initial steady-state adjustment rate equal to 0.2.[10] Figure 5 plots the economy's response to an unanticipated permanent 10 percentage decrease in τ^k. We find that both the short- and long-runs effects in this case are much larger than in Figure 4. The main reason is due to the larger extensive margin effect. In particular, the adjustment rate rises by 10 and 20 percent on impact for the Lumpy1 and Lumpy2 models.

While the initial high adjustment rate constrains the effectiveness of an expansionary tax policy, it makes a contractionary tax policy more effective. To illustrative this point, Figure 6 presents the economy's response to an unanticipated permanent 10 percentage point increase in τ^k. This figure reveals that for Lumpy2 model, the adjustment rate decreases by about 18

[10] We recalibrate $\xi_{\max}$ in the Lumpy1 and Lumpy2 models.

percent immediately, causing the aggregate investment rate falls by about 20 percent immediately.

When the permanent tax cut is initially anticipated to be enacted in period 5, the economy's short-run response is different from the unanticipated tax cut case, even though they give the same long-run steady state. Figure 7 presents the transition dynamics. We find that the short-run effect of the expected permanent tax cut is smaller than that of the unexpected permanent tax cut. We also find that the investment rate rises initially and continues to rise until period 4. For the two lumpy investment models, the adjustment rate also rises initially and continue to rise until period 4. Starting from period 5, the investment rate and the adjustment rate decrease monotonically to their steady state values.

4.4 Temporary Changes in the Investment Tax Credit

Suppose a 10 percent ITC is imposed from periods 1-4 unexpectedly. In period 5 this policy is repealed. Figure 8 plots the economy's dynamic responses. The ITC makes investment cheaper and hence reduces the price of capital Q. Investment jumps immediately and then decreases until period 4. In period 5 investment drops sharply below its steady state level because the ITC is removed in period 5. After period 5, investment gradually rises to its previous steady-state level. Consumption follows the opposite pattern.

For the two lumpy investment models (Lumpy1 and Lumpy2), the adjustment rate rises by 10 and 20 percent respectively on impact and then decreases until period 4. It drops below its steady state level in period 5. After period 5, it gradually rises to its previous steady-state level. Even though this extensitive margin effect is large (accounting half of the increase in the aggregate investment rate), the initial rise of investment rate is smaller in the two lumpy investment models than the frictionless RBC model. This result is different from the one in the case of corporate income tax changes. The intuition is the following. The presence of convex adjustment costs in the partial adjustment model makes the impact response of investment smaller than that in the RBC model. Introducing fixed adjustment costs to the partial adjustment model adds an extensive margin effect, leading to a larger response to ITC. However, the extensive margin effect is not large enough to make the response in the lumpy investment model larger than that in the RBC model, because the decrease in Q reduces firm profitability and hence dampens the positive extensive margin effect.

Next, we suppose the enactmanet of ITC in period 5 is anticipated in period 1. The 10 percent ITC lasts from periods 5 to 8. Figure 9 presents the economy's dynamic response to this tax policy. Contrary to the responses presented in Figure 6, investment and output decrease, but consumption increases on impact. The intuition comes from the dynamics of the price of capital Q. We use the lumpy investment model characterized in Proposition 2 to explain the intuition. Anticipating the fall of Q in period 5 due to ITC, Q falls immediately at date 1. Because the investment rate is determined by equation (25) and $\tau_t^i = 0$ for $t = 1, ..., 4$, the investment rate must decrease from periods 1 to 4. In period 5, the ITC makes the new investment good cheaper and hence $Q_t / \left(1 - \tau_t^i\right)$ actually rises. Thus, the investment rate jumps up in period 5. Starting from period 5, the economy's response is similar to that in the case of unexpected temporary increase in the ITC.

4.5 Permanent Changes in the Investment Tax Credit

We finally consider two experiments in which there is a permanent 10 percent increase in the ITC. Figure 10 presents the economy's response when this tax change is unexpectedly enacted initially. In period 1, the investment rate rises immediately, but the increase is less than that if the ITC is temporary, as shown in Figure 8. This is in sharp contrast to Abel's (1982) result that a temporary ITC provides a greater stimulus to investment than a permanent ITC, except for a competitive firm with constant returns to scale. The reason is that Abel (1982) uses a partial equilibrium model rather than a general equilibrium model. As we point out in Section 4.2, in partial equilibrium Q can be determined independent of capital with competitive firms and constant-returns-to-scale technology. We can then show that the initial rise of the investment rate is independent of the duration of the ITC. By contrast, in general equilibrium, Q and capital must be jointly determined. As Figure 8 and 10 show, the initial fall in Q is larger in response to the permanent increase in the ITC than in response to the temporary increase in the ITC.

A permanent increase in the ITC changes the economy's steady state. For all four models (RBC, PA, Lumpy1 and Lumpy2), the capital stock, ouput, consumption, and labor in the new steady state are about 25, 9, 8, and 2 percent, respectively, higher than in the initial steady state. But the steady-state adjustment rate in the two lumpy investment models is lower than their initial steady-state values, causing the steady-state aggregate investment to be lower than that in the frictionless RBC and partial adjustment models. Because Q decreases in the steady state, in the long run firms make less investment and less firms make investment in the presence of fixed adjustment costs.

When the permanent increase in the ITC is expected initially to be enacted in period 5, firms respond by decreasing investment initially. This result is similar to that in the case of expected temporary increase in the ITC. Figure 11 presents the transition dynamics. This figure reveals that consumption in period 5 drops sharply for all four models in order for firms to raise investment by taking the benefit of ITC.

4.6 Welfare Effects of Tax Policy

Our general equilibrium model allows us to conduct welfare analysis. To evaluate the welfare effect, it is important to make the distinction between the utility gain in the steady state and the utility gain in the transition path. For temporary tax changes, the economy's steady state often remains unchanged. Therefore, there is no weflare effect in the steady state. However, in the transition path following a tax change, the equilibrium consumption and leisure may take values different from their steady-state values. Thus, the indirect lifetime utility level derived from these equilibrium consumption and leisure streams after the tax change may be different from that in the equilibrium before the tax change. In the following analysis, we will focus on the welfare effect during transition rather than in the steady state.

We measure the welfare gain following a tax policy using the percentage consumption increase. Specifically, let the indirect life-time utility level before a tax change be given by:

$$U^b = \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t^b\right) - \varphi N_t^b \right]$$

where C_t^b and N_t^b denote the equilibrium consumption and labor in period t before the tax change. Let U^a denote the indirect life-time utility level in the equilibrium after the tax change. The welfare gain Δ is defined by the following equation:

$$U^a = \sum_{t=0}^{\infty} \beta^t \left[\log\left((1+\Delta)\, C_t^b\right) - \varphi N_t^b \right].$$

Solving the above equation yields:

$$\Delta = \exp\left[(1-\beta)\left(U^b - U^a\right)\right] - 1.$$

We use this formula to compute welfare gains for our previous 8 tax policy experiments. Table 2 presents the results.

Table 2 reveals several interesting results. First, the welfare effects for all four models are similar and generally small. The welfare benefits in the lumpy investment model are generally higher than those in the partial adjustment model. In addition, the distribution of firms matter for the calculation of welfare benefits because this distribution determines the extensive margin effect in the presence of fixed capital adjustment costs. As we analyzed earlier, the Lumpy2 model generates a higher extensive margin effect than the Lumpy1 model, causing the welfare gains in the Lumpy2 model to be larger than those in the Lumpy1 model. Second, the welfare gains are larger when the temporary corporate income tax cut is expected than when it is unexpected. But there is no anticipation benefit when the corporate income tax cut is permanent. Third, the welfare benefits from a permanent increase in the ITC are larger than those from a temporary increase in the ITC. In addition, there is a welfare gain if the increase in the ITC is unexpected rather than expected, contrary to the case of the temporary decrese in the corporate income tax rate. The last two results are consistent with Judd's (1987) findings in a continuous-time model without capital adjustment costs. The intuition is that, regardless the presence of capital adjustment costs, the decrease in the future capital income tax rate raises investment immediately, while the increase in the future ITC reduces investment initially.

Table 2. Welfare gains from the tax changes.

	RBC	PA	Lumpy1	Lumpy2
Temporary unexpected $\tau_t^k \downarrow$	0.06	0.04	0.06	0.08
Temporary expected $\tau_t^k \downarrow$	0.12	0.09	0.14	0.18
Permanent unexpected $\tau_t^k \downarrow$	1.02	0.93	1.25	1.45 (2.95, C3.75, L0.98)
Permanent expected $\tau_t^k \downarrow$	0.97	0.90	1.20	1.40
Temporary unexpected $\tau_t^i \uparrow$	0.55	0.40	0.46	0.51
Temporary expected $\tau_t^i \uparrow$	0.33	0.31	0.32	0.32
Permanent unexpected $\tau_t^i \uparrow$	2.42	2.22	2.33	2.35
Permanent expected $\tau_t^i \uparrow$	1.85	1.83	1.87	1.83

Note: This table presents welfare gains in percentage for the 8 policy experiments studied in Section 4.

5 Conclusion

Appendix

A Proofs

Proof of Proposition 1: From (21), we can show that the target investment level i_t^j satisfies the first-order condition:

$$1 - \tau_t^i = g'\left(i_t^j\right) \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1}. \tag{A.1}$$

By equations (3), (17) and (20), we can derive equation (23). Using this equation, we define $V_t^a\left(\xi_t^j\right)$ as the price of capital when the firm chooses to invest. It is given by:

$$\begin{aligned} V_t^a\left(\xi_t^j\right) &= \left(1 - \tau_t^k\right) R_t + \tau_t^k \delta - \left(1 - \tau_t^i\right) i_t^j - \xi_t^j + g(i_t^j) Q_t \\ &= \left(1 - \tau_t^k\right) R_t + \tau_t^k \delta + (1 - \delta + \varsigma) Q_t \\ &\quad + \frac{\theta}{1-\theta} (\psi Q_t)^{\frac{1}{\theta}} \left(1 - \tau_t^i\right)^{\frac{\theta - 1}{\theta}} - \xi_t^j. \end{aligned} \tag{A.2}$$

Define V_t^n as the price of capital when the firm chooses not to invest. It satisfies:

$$V_t^n = \left(1 - \tau_t^k\right) R_t + \tau_t^k \delta + (1 - \delta + \varsigma) Q_t, \tag{A.3}$$

which is independent of ξ_t^j. We can then rewrite problem (21) as:

$$V_t\left(\xi_t^j\right) = \max\left\{V_t^a\left(\xi_t^j\right), V_t^n\right\}. \tag{A.4}$$

Clearly, there is a unique cutoff value ξ_t^* given in (22) satisfying the condition:

$$V_t^a\left(\xi_t^*\right) = V_t^n, \tag{A.5}$$

$$V_t^a\left(\xi_t^j\right) > V_t^n \text{ if and only if } \xi_t^j < \xi_t^*. \tag{A.6}$$

Because the support of ξ_t^j is $[0, \xi_{\max}]$, the investment trigger is given by $\bar{\xi}_t \equiv \min\{\xi_t^*, \xi_{\max}\}$.

We can show that:

$$\begin{aligned} \bar{V}_t &= \int_0^{\xi_{\max}} V_t(\xi) \phi(\xi) d\xi \\ &= \int_{\bar{\xi}_t}^{\xi_{\max}} V_t^n \phi(\xi) d\xi + \int_0^{\bar{\xi}_t} V_t^a(\xi) \phi(\xi) d\xi \\ &= V_t^n + \int_0^{\bar{\xi}_t} \left[V_t^a(\xi) - V_t^n\right] \phi(\xi) d\xi. \end{aligned}$$

We use equations (A.2), (A.3) and (22) to derive

$$\begin{aligned} V_t^a(\xi) - V_t^n &= \frac{\theta}{1-\theta} (\psi Q_t)^{\frac{1}{\theta}} \left(1 - \tau_t^i\right)^{\frac{\theta-1}{\theta}} - \xi \\ &= \xi_t^* - \xi. \end{aligned} \tag{A.7}$$

Using the above two equations, (A.3), and (20), we obtain (24). Q.E.D.

Proof of Proposition 2: From (13), we deduce that all firms choose the same labor-capital ratio n_t. We thus obtain $N_t = n_t K_t$. We then derive

$$\begin{aligned} Y_t &= \int Y_t^j dj = \int F\left(K_t^j, N_t^j\right) dj = \int F\left(1, n_t^j\right) K_t^j dj \\ &= F(1, n_t) \int K_t^j dj = F(1, n_t) K_t = F(K_t, N_t), \end{aligned}$$

which gives the first equality in equation (27). As a result, we use equation (13) and $n_t^j = n_t$ to show:

$$F_2(K_t, N_t) = (1 - \tau_t^n) w_t. \tag{A.8}$$

By the constant return to scale property of F, we also have:

$$R_t = F_1(K_t, N_t). \tag{A.9}$$

Equation (**??**) follows from equation (22) and (9). We next derive aggregate investment:

$$I_t = \int I_t^j dj = \int i_t^j K_t^j dj = K_t \int_0^{\bar{\xi}_t} (\psi Q_t)^{\frac{1}{\theta}} \phi(\xi) d\xi,$$

where the second equality uses the definition of i_t^j, the third equality uses a law of large numbers and the optimal investment rule (23). We thus obtain (25).

We turn to the law of motion for capital. By definition,

$$K_{t+1} = \int_0^1 \left[(1-\delta) + \Phi(i_t^j)\right] K_t^j dj.$$

Substituting the optimal investment rule in equation (23) and using equation (25), we obtain (26).

Equation (29) follows from substituting equations (9) and (A.9) into equation (24). Equation (28) follows from equations (9), (10) and (A.8). Finally, equation (27) follows from a law of large number, the market clearing condition (12), and Proposition 1. Q.E.D.

Proof of Proposition 3: In an interior steady state, $\xi^* = \bar{\xi}$ and equations (25) and (22) imply that:

$$\frac{I}{K} = \left(\frac{\psi Q}{1-\tau^i}\right)^{\frac{1}{\theta}} \int_0^{\bar{\xi}} \phi(\xi) d\xi, \tag{A.10}$$

$$\bar{\xi} = \frac{\theta}{1-\theta} (\psi Q)^{\frac{1}{\theta}} \left(1-\tau^i\right)^{\frac{\theta-1}{\theta}}, \tag{A.11}$$

From these two equations, we obtain:

$$\frac{I}{K} = \frac{\bar{\xi}}{1-\tau^i} \frac{1-\theta}{\theta} \int_0^{\bar{\xi}} \phi(\xi) d\xi. \tag{A.12}$$

In steady state, equation (26) becomes:

$$\delta - \varsigma = \frac{\psi}{1-\theta} (I/K)^{1-\theta} \left[\int_0^{\bar{\xi}} \phi(\xi) d\xi\right]^{\theta}. \tag{A.13}$$

Substituting equation (A.12) into the above equation yields equation (31). The expression on the right-hand side of this equation increases with $\bar{\xi}$. The condition in this proposition guarantees a unique interior solution $\bar{\xi} \in (0, \xi_{\max})$ exists.

Equation (32) follows from (A.11). Equations (A.12) and (A.13) imply that:

$$\delta - \varsigma = \frac{\psi}{1-\theta} \frac{I}{K} \left(\frac{\xi^* (1-\theta)}{(1-\tau^i)\,\theta}\right)^{-\theta}. \tag{A.14}$$

From this equation and equation (32), we obtain (33). The other equations in the proposition follow from the steady-state versions of equations (28)-(29). Q.E.D.

References

Abel, Andrew B. and Janice C. Eberly, 1996, "Optimal investment with costly irreversibility," *Review of Economic Studies* 63, 581-593.

Abel, Andrew B. and Janice C. Eberly, 1998, "The Mix and Scale of Factors with Irreversibility and Fixed Costs of Investment," in Bennett McCallum and Charles Plosser (eds.) *Carnegie-Rochester Conference Series on Public Policy* 48, 101-135.

Bachmann, Ruediger; Caballero, Ricardo, and Engel, Eduardo. 2008. "Aggregate Implications of Lumpy Investment: New Evidence and a DSGE Model." NBER working paper No. 12336.

Baxter, Marianne and Mario J. Crucini, 1993, “Explaining Saving-Investment Correlations,” *American Economic Review* 83, 416-436.

Caballero, Ricardo and Engel, Eduardo. 1999. “Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S,s) Approach.” Econometrica, 67(4), pp. 783-826.

Caballero, Ricardo; Engel, Eduardo and Haltiwanger, John. 1995. “Plant-Level Adjustment and Aggregate Investment Dynamics.” Brookings Papers on Economic Activity, Vol. 1995, No. 2, pp. 1-54.

Caballero, Ricardo, and Leahy, John V. 1996. “Fixed Costs: The Demise of Marginal q.” NBER working paper No. 5508.

Cooper, Russell and Haltiwanger, John. 2006. “On the Nature of Capital Adjustment Costs.” Review of Economic Studies, 73, pp. 611-634.

Cooper, Russell, Haltiwanger, John and Power, Laura. 1999. “Machine Replacement and the Business Cycle: Lumps and Bumps.” American Economic Review 89, 921-946.

Doms, Mark and Dunne, Timothy. 1998. “Capital Adjustment Patterns in Manufacturing Plants.” Review of Economic Dynamics, 1, pp. 409-429.

Dotsey, M.; King, R. and Wolman, A. 1999. “State Dependent Pricing and the General Equilibrium Dynamics of Money and Output,” Quarterly Journal of Economics, 104, 655-690.

Fisher, Jonas D.M. 2006. “The Dynamic Effects of Neutral and Investment-Specific Technology Shocks.” Journal of Political Economy, June 2006, 114(3), pp. 413-452.

Gourio, Francois and Kashyap, Anil. 2007. “Investment Spikes: New Facts and a General Equilibrium Exploration.” Journal of Monetary Economics, 54: Supplement 1, pp. 1-22.

Greenwood, J., Hercowitz, Z., and P. Krusell, 2000, “The role of investment-specific technological change in the business cycle,” European Economic Review 44, 91-115.

Hayashi, Fumio. 1982. “Tobin’s Marginal q and Average q: A Neoclassical Interpretation.” Econometrica. 50, January, pp. 213-224.

House, Christopher L., 2008, “Fixed Costs and Long-Lived Investments,” working paper, University of Michigan.

Jermann, Urban J., 1998, "Asset Pricing in Production Economies," *Journal of Monetary Economics* 41, 257-275.

Khan, Aubhik and Thomas, Julia. 2003. "Nonconvex Factor Adjustments in Equilibrium Business Cycle Models: Do Nonlinearities Matter?" Journal of Monetary Economics, 50, pp. 331-360.

Khan, Aubhik and Thomas, Julia. 2008. "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics." Econometrica, 76(2), pp. 395-436.

King, Robert G., Charles I. Plosser, and Sergio T. Rebelo, 2002, "Production, Growth and Business Cycles: Technical Appendix," *Computational Economics* 20, 87-116.

Krusell, P., and Smith, A. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." Journal of Political Economy 106, 867-898.

Miao, Jianjun, 2008, "Corporate Tax Policy and Long-Run Capital Formation: The Role of Irreversibility and Fixed Costs," working paper, Boston University.

Prescott, Edward C., 1986, "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneapolis Quarterly Review* 10, 9-22.

Thomas, Julia. 2002. "Is Lumpy Investment Relevant for the Business Cycle?" Journal of Political Economy, 110, 508-534.

Uzawa, Hirofumi, 1969, "Time Preference and the Penrose Effect in a Two-Class Model of Economic Growth," *Journal of Political Economy* 77, 628-52.

Veracierto, Marcelo. L. 2002. "Plant-Level Irreversible Investment and Equilibrium Business Cycles." American Economic Review, 92, 181-197.

Wang, Pengfei and Yi Wen, 2009, "Financial Deveolpment and Economic Volatility: A Unified Explanation," working paper, Hong Kong University of Science and Technology.

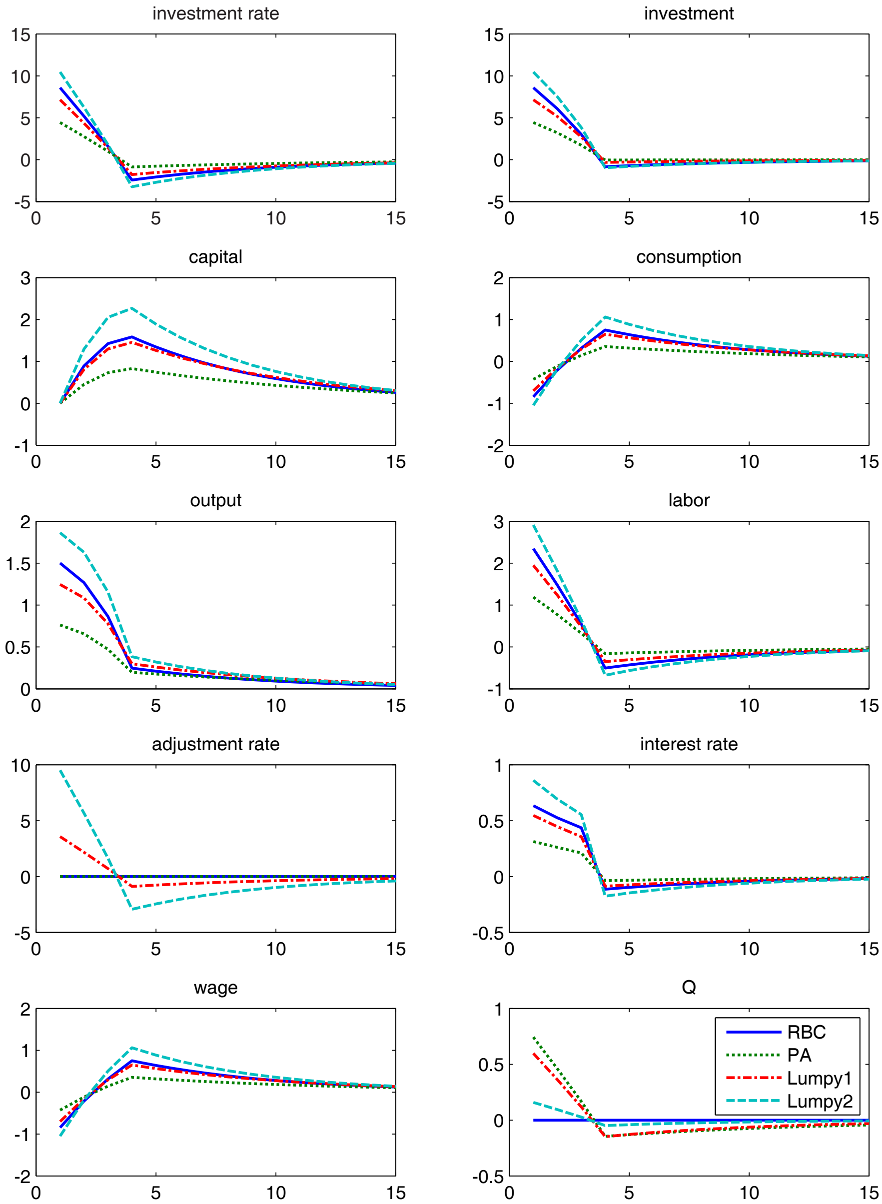

Figure 1: **Response to unexpected temporary decrease in τ^k.** The tax cut lasts from periods 1 to 4. The vertical axis measures the percentage deviation from the initial steady state. The horizontal axis measures time.

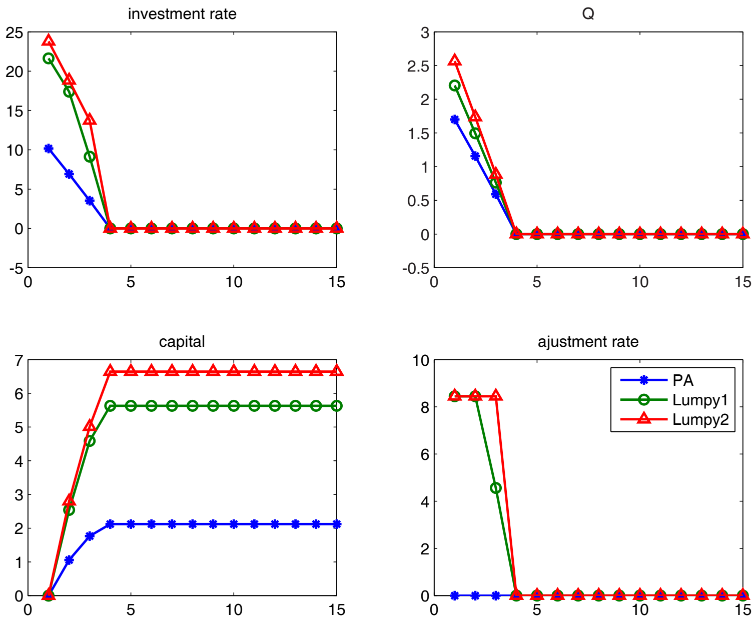

Figure 2: **Response to unexpected temporary decrease in τ^k in partial equilibrium.** The tax cut lasts from periods 1 to 4. The vertical axis measures the percentage deviation from the initial steady state. The horizontal axis measures time.

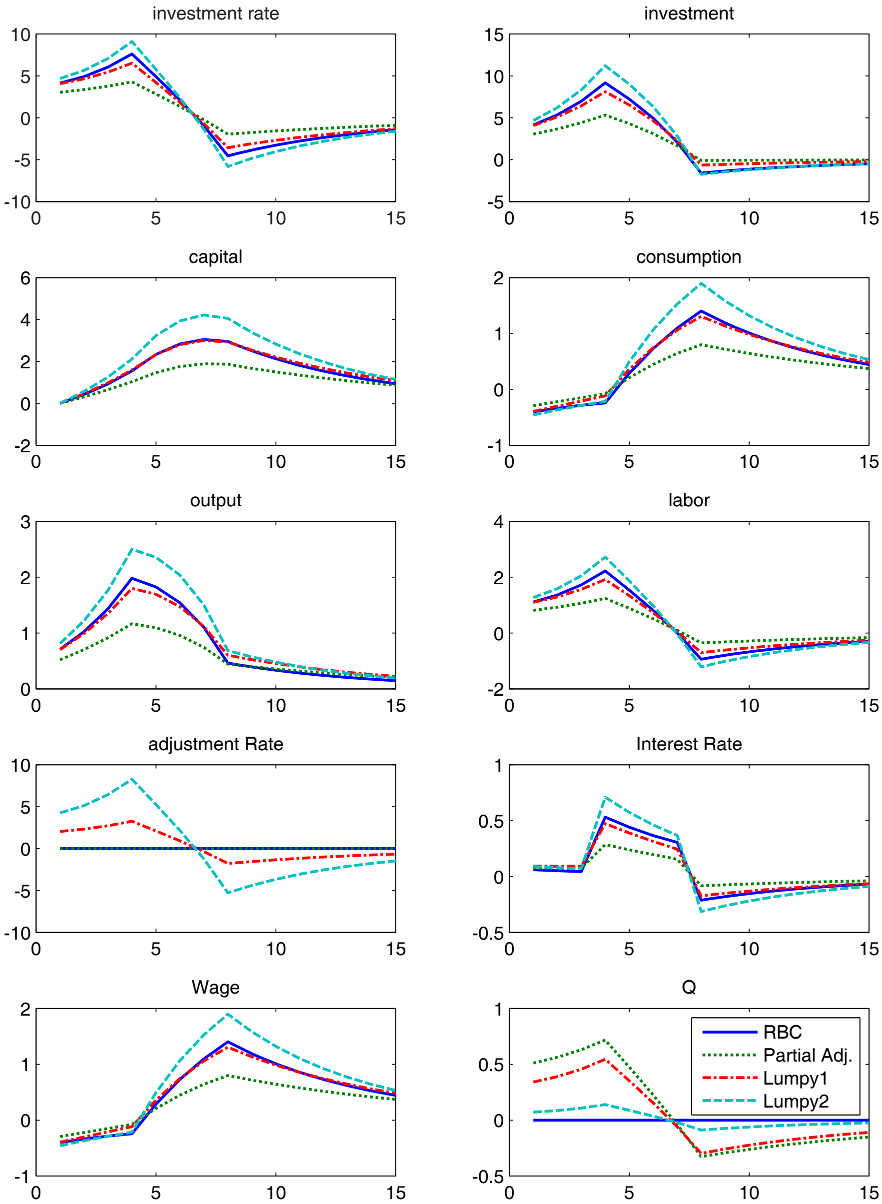

Figure 3: **Response to expected temporary decrease in τ^k.** The tax cut lasts from periods 5 to 8. The vertical axis measures the percentage deviation from the initial steady state. The horizontal axis measures time.

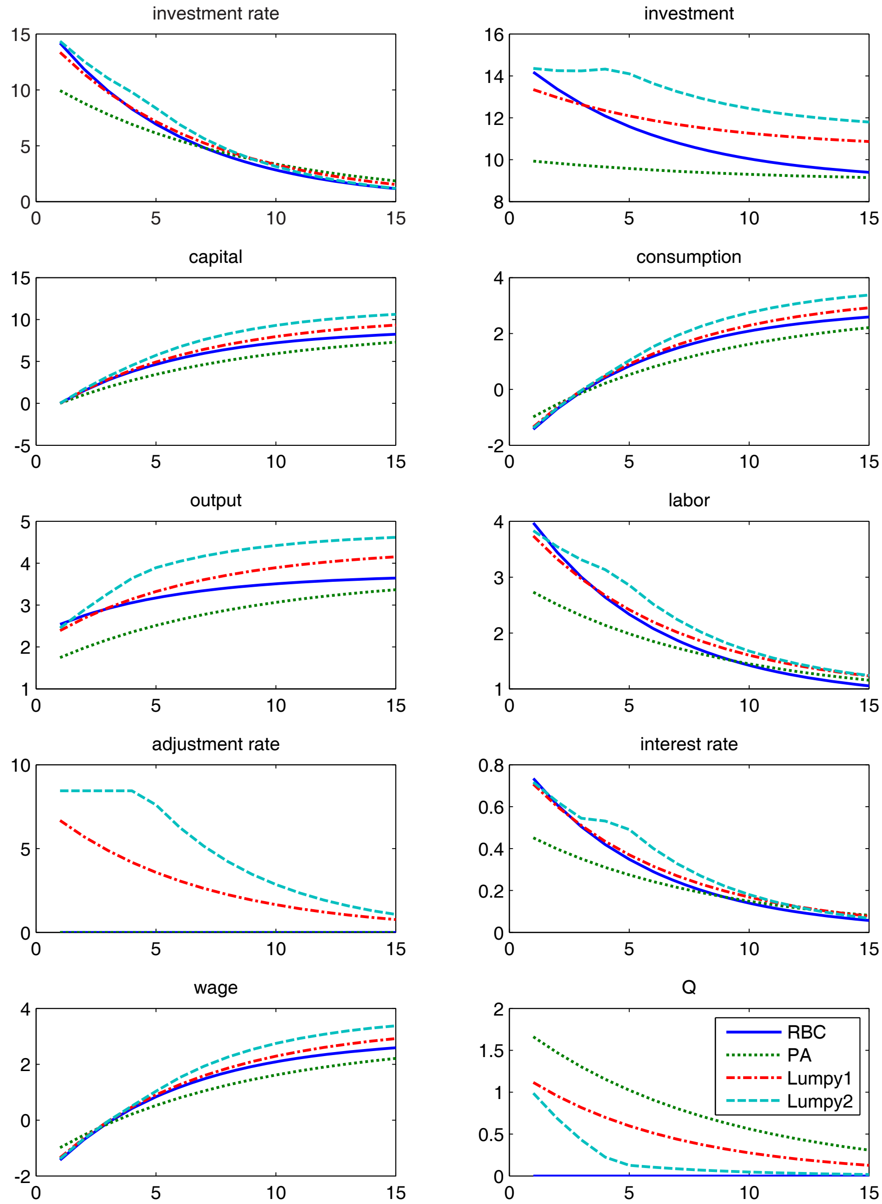

Figure 4: **Response to unexpected permanent decrease in τ^k.** The vertical axis measures the percentage deviation from the initial steady state. The horizontal axis measures time.

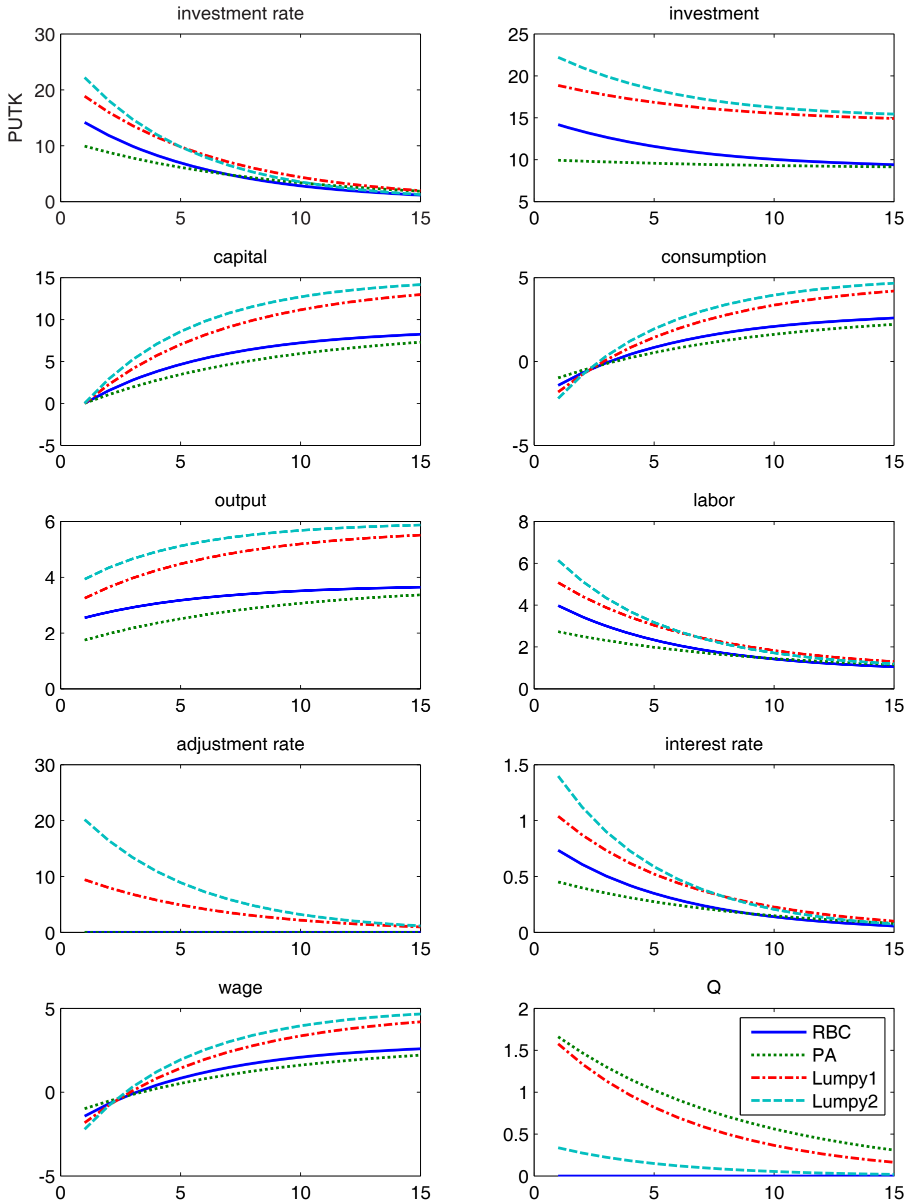

Figure 5: **Response to unanticipated permanent decrease in τ^k when the initial adjustment rate is 0.8.** The vertical axis measures the percentage deviation from the initial steady state. The horizontal axis measures time.

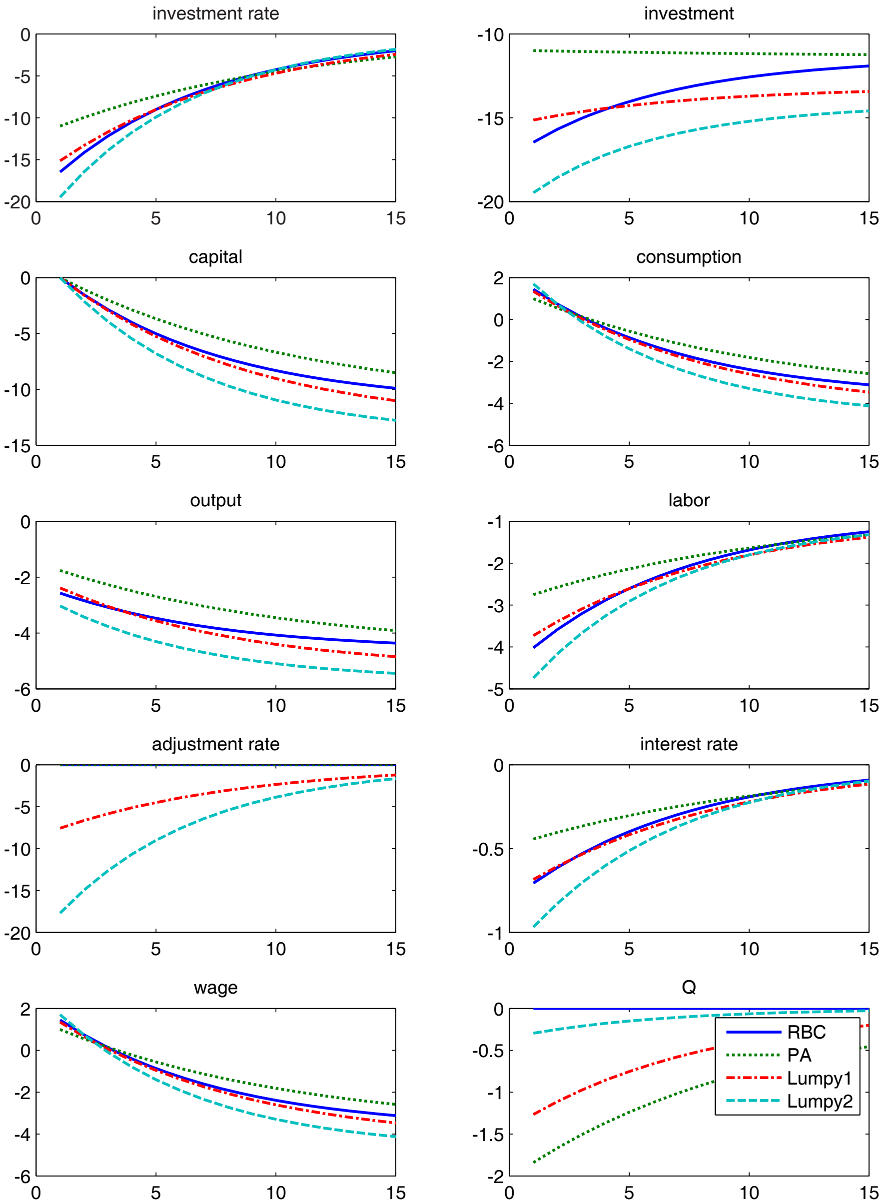

Figure 6: **Response to unanticipated permanent increase in τ^k.** The vertical axis measures the percentage deviation from the initial steady state. The horizontal axis measures time.

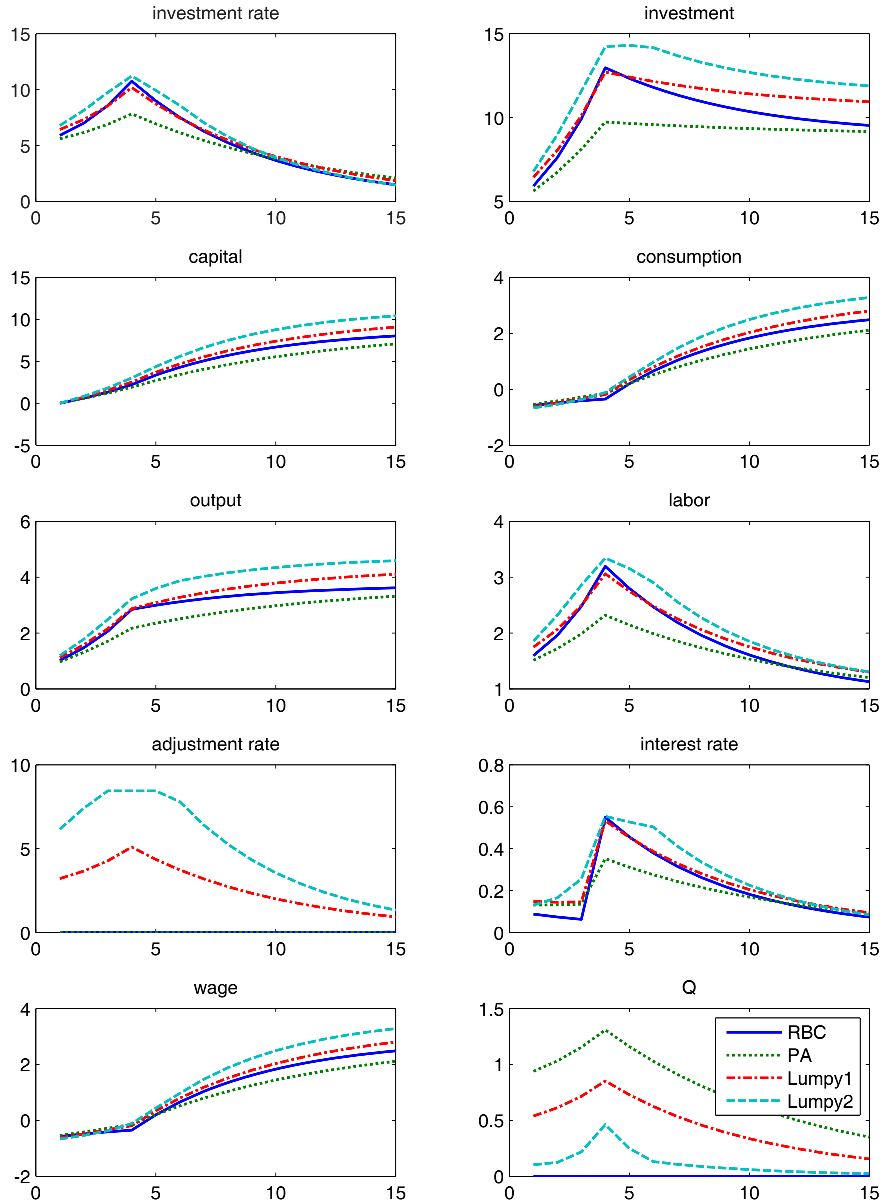

Figure 7: **Response to anticipated permanent decrease in τ^k.** The tax cut is enacted in period 5. The vertical axis measures the percentage deviation from the initial steady state. The horizontal axis measures time.

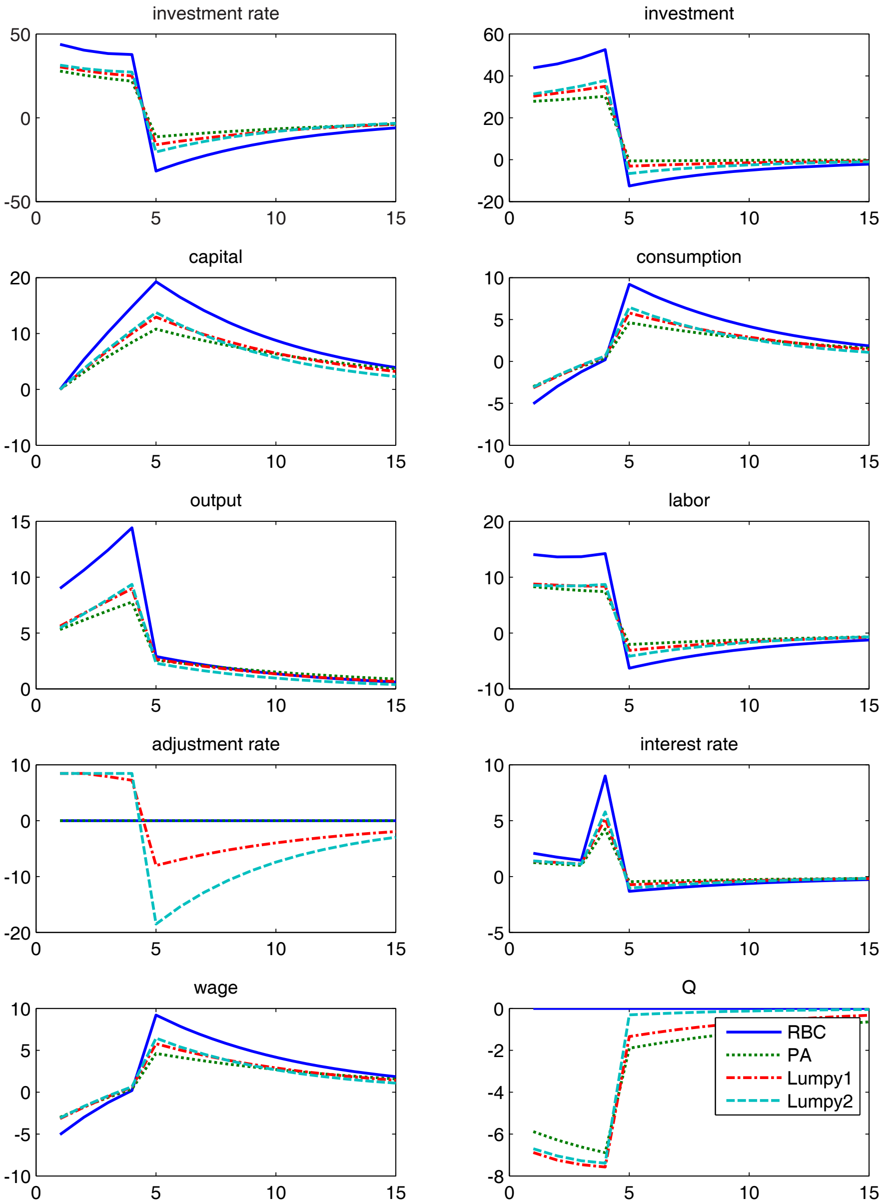

Figure 8: **Response to unanticipated temporary increase in τ^i.** The tax increase lasts from periods 1 to 4. The vertical axis measures the percentage deviation from the initial steady state. The horizontal axis measures time.

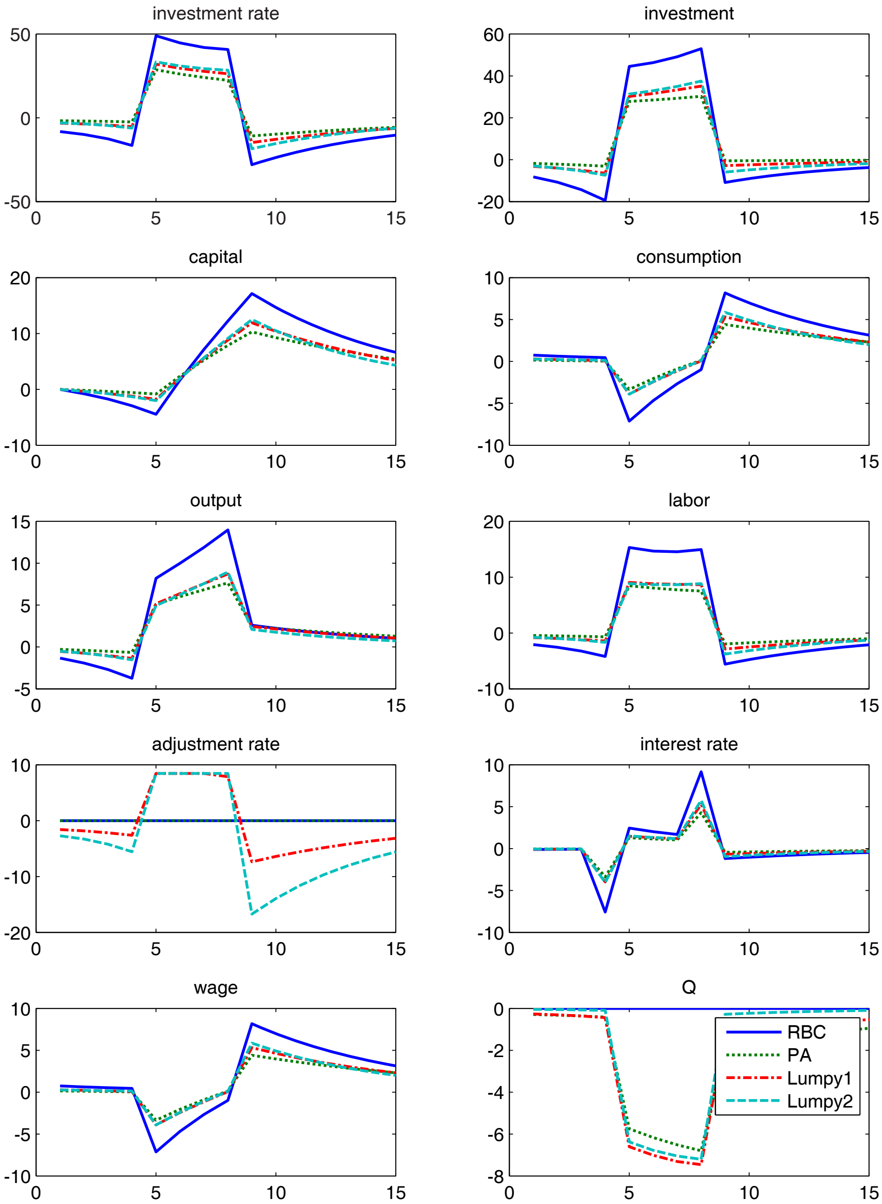

Figure 9: **Response to anticipated temporary increase in τ^i.** The tax increase lasts from periods 5 to 8. The vertical axis measures the percentage deviation from the initial steady state. The horizontal axis measures time.

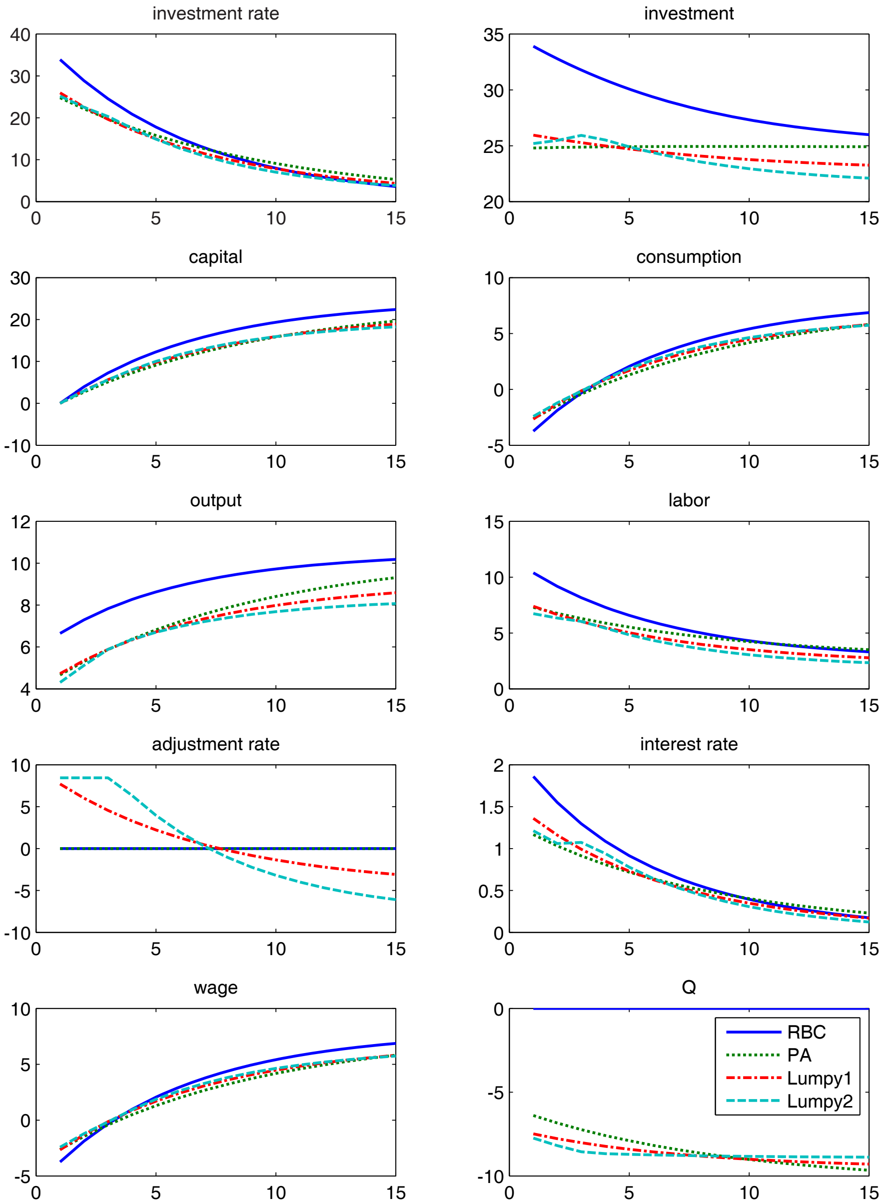

Figure 10: **Response to unanticipated permanent increase in τ^i.** The vertical axis measures the percentage deviation from the initial steady state. The horizontal axis measures time.

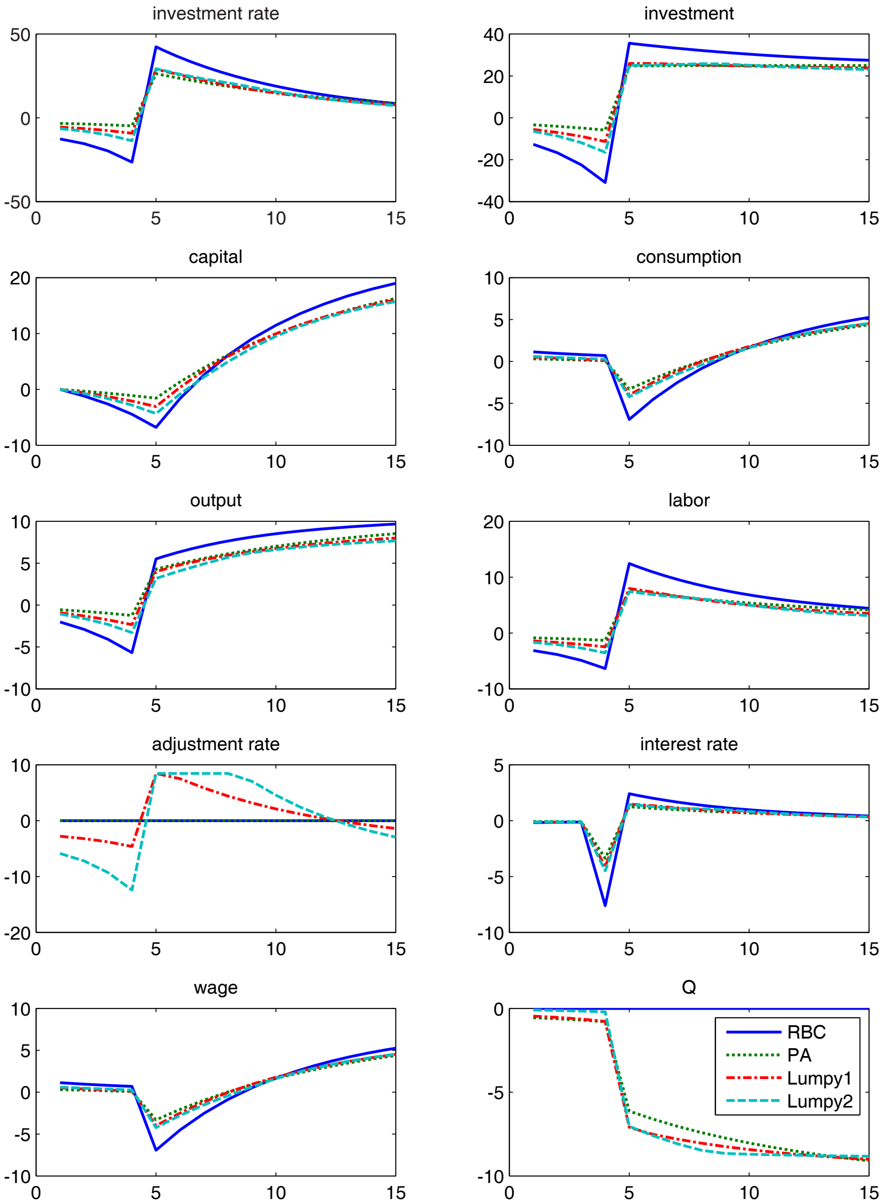

Figure 11: **Response to anticipated permanent increase in τ^i.** The tax increase is enacted in period 5. The vertical axis measures the percentage deviation from the initial steady state. The horizontal axis measures time.